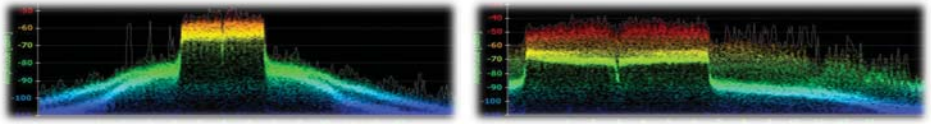

ECS455: Chapter 5

OFDM

5.3 OFDM as Multi-Carrier Transmission

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Single-Carrier Digital Transmission

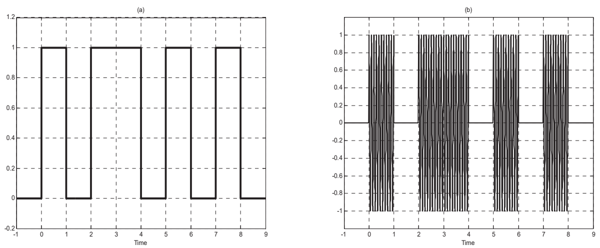
- Baseband:

$$s(t)=\sum_{k=0}^{N-1} s_k\, p(t-kT_s)$$

$$p(t)=1_{[0,T_s)}(t)=\begin{cases}1, & t\in[0,T_s)\\ 0, & \text{otherwise.}\end{cases}$$

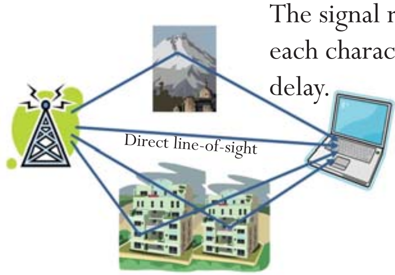
- Passband:

$$x(t)=\mathrm{Re}\left\{s(t)e^{j2\pi f_c t}\right\}$$



Wireless Comm. and Multipath Fading

The signal received consists of a number of reflected rays, each characterized by a different amount of attenuation and delay.

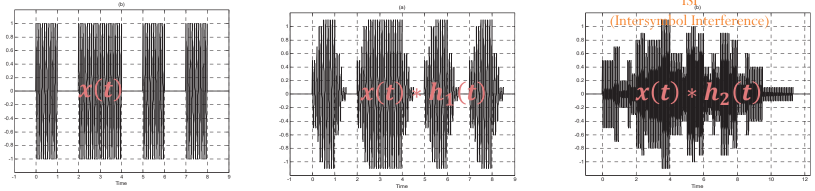


$$y(t)=x(t)*h(t)+n(t)=\sum_{i=0}^{v}\beta_i x(t-\tau_i)+n(t)$$

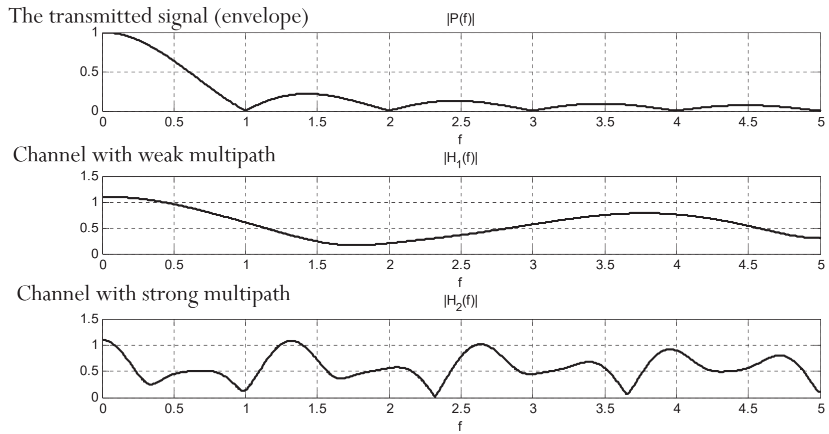
$$h(t)=\sum_{i=0}^{v}\beta_i\delta(t-\tau_i)$$

$$h_1(t)=0.5\delta(t)+0.2\delta(t-0.2T_s)+0.3\delta(t-0.3T_s)+0.1\delta(t-0.5T_s)$$

$$h_2(t)=0.5\delta(t)+0.2\delta(t-0.7T_s)+0.3\delta(t-1.5T_s)+0.1\delta(t-2.3T_s)$$



Frequency Domain

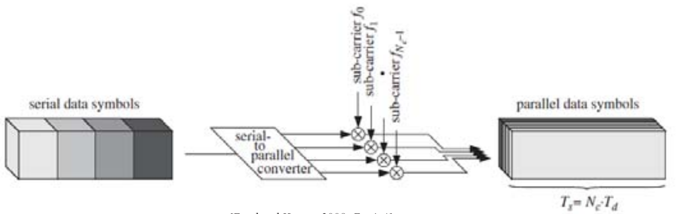


Observation and a Solution

- Observation: Delay spread causes ISI
- A general rule of thumb is that a delay spread of less than 5 or 10 times the symbol width will not be a significant factor for ISI.
- Solution: The ISI can be mitigated by reducing the symbol rate and/or including sufficient guard times between symbols.

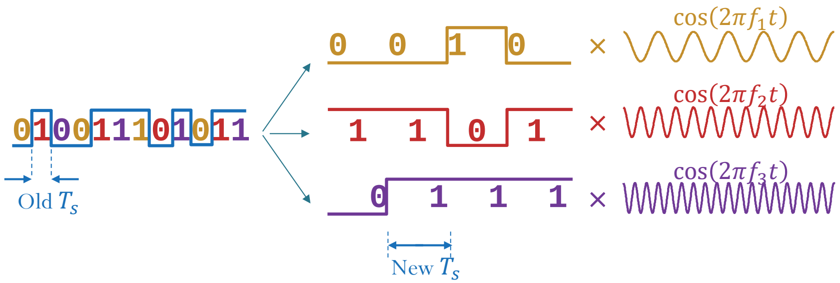
Multi-Carrier Transmission

- Convert a serial high rate data stream on to **multiple parallel low rate** sub-streams.
- Each sub-stream is modulated on its own **sub-carrier**.
- **Time domain perspective**: Since the symbol rate on each sub-carrier is much less than the initial serial data symbol rate, the effects of delay spread, i.e. ISI, significantly decrease, reducing the complexity of the equalizer.



[Fazel and Kaiser, 2008, Fig 1-4]

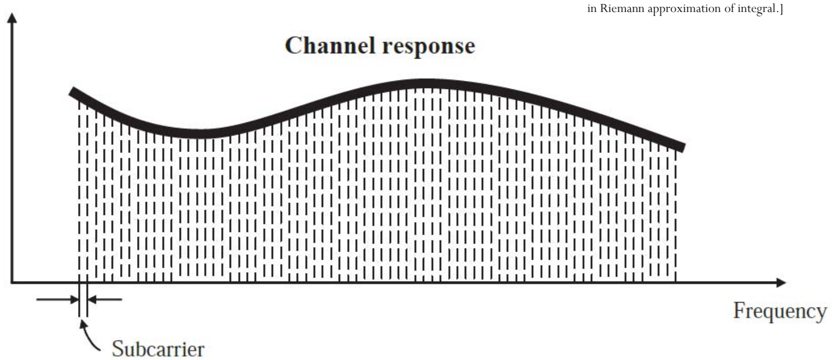
Multi-Carrier Modulation



Frequency Division Multiplexing

- **Frequency Domain Perspective**: Even though the fast fading is frequency-selective across the entire OFDM signal band, it is effectively flat in the band of each low-speed signal.

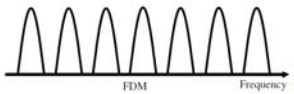
[The flatness assumption is the same one that you used in Riemann approximation of integral.]



[Myung and Goodman, 2008]

Frequency Division Multiplexing (FDM)

- To facilitate separation of the signals at the receiver, the carrier frequencies were **spaced sufficiently far apart** so that the signal spectra did not overlap. Empty spectral regions between the signals assured that they could be separated with readily realizable filters.
- The resulting spectral efficiency was therefore quite low.

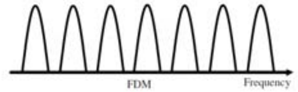


Single Carrier vs. Multi-Carrier (FDM)

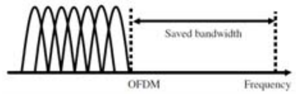
Single Carrier	Multi-Carrier (FDM)
Single higher rate serial scheme	Parallel scheme. Each of the parallel subchannels can carry a low signaling rate, proportional to its bandwidth.
❍ Multipath problem: Far more susceptible to inter-symbol interference (ISI) due to the short duration of its signal elements and the higher distortion produced by its wider frequency band ❍ Complicated equalization	❍ Long duration signal elements and narrow bandwidth in sub-channels. ❍ Complexity problem: If built straightforwardly as several (N) transmitters and receivers, will be more costly to implement. ❍ BW efficiency problem: The sum of parallel signalling rates is less than can be carried by a single serial channel of that combined bandwidth because of the unused guard space between the parallel sub-carriers.

OFDM

- OFDM = **Orthogonal** frequency division multiplexing
- One of multi-carrier modulation (MCM) techniques
  - Parallel data transmission (of many sequential streams)
  - A broadband is divided into many narrow sub-channels
  - Frequency division multiplexing (FDM)



- High spectral efficiency
  - The sub-channels are made **orthogonal** to each other over the **OFDM symbol duration T_s**.
    - Spacing is carefully selected.
  - Allow the sub-channels to overlap in the frequency domain.
  - Sub-carriers are spaced as close as theoretically possible.



Baseband OFDM Symbol

- Let $\underline{\mathbf{S}} = (S_1, S_2, \ldots, S_N)$ be the information vector.
- One baseband OFDM modulated symbol can be expressed as

$$s(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k \exp\left(j\frac{2\pi kt}{T_s}\right), \quad 0 \le t \le T_s$$

$$= \sum_{k=0}^{N-1} S_k \underbrace{\frac{1}{\sqrt{N}} 1_{[0,T_s]}(t) \exp\left(j\frac{2\pi kt}{T_s}\right)}_{c_k(t)}$$

Some references may use different constant in the front

Some references may start with different time interval, e.g. [$-T_s/2$, $+T_s/2$]

Note that:

$$\mathrm{Re}\{s(t)\} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \left(\mathrm{Re}\{S_k\} \cos\left(\frac{2\pi kt}{T_s}\right) - \mathrm{Im}\{S_k\} \sin\left(\frac{2\pi kt}{T_s}\right) \right)$$

OFDM and CDMA: Waveform Version

- Recall: Orthogonality-Based MA (CDMA)

$$s(t)=\sum_{k=0}^{\ell-1} S_k c_k(t) \quad \text{where} \quad c_{k_1} \perp c_{k_2}$$

- Baseband OFDM modulated symbol:

$$\begin{aligned} s(t) &= \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} S_k \exp\left(j\frac{2\pi kt}{T_s}\right), \quad 0\le t\le T_s \\ &= \sum_{k=0}^{N-1} S_k \underbrace{\frac{1}{\sqrt{N}} 1_{[0,T_s]}(t)\exp\left(j\frac{2\pi kt}{T_s}\right)}_{c_k(t)} \end{aligned}$$

Another "special case" of CDMA!

OFDM: Orthogonality

$$\begin{aligned} \int c_{k_1}(t) c_{k_2}^*(t)dt &= \int_0^{T_s} \exp\left(j\frac{2\pi k_1 t}{T_s}\right)\exp\left(-j\frac{2\pi k_2 t}{T_s}\right)dt \\ &= \int_0^{T_s} \exp\left(j\frac{2\pi (k_1-k_2) t}{T_s}\right)dt = \begin{cases} T_s, & k_1 = k_2 \\ 0, & k_1 \ne k_2 \end{cases} \end{aligned}$$

When $k_1 = k_2$,

$$\int c_{k_1}(t) c_{k_2}^*(t)dt = \int_0^{T_s} 1dt = T_s$$

When $k_1 \ne k_2$,

$$\begin{aligned} \int c_{k_1}(t) c_{k_2}^*(t)dt &= \frac{T_s}{j2\pi(k_1-k_2)}\exp\left(j\frac{2\pi (k_1-k_2) t}{T_s}\right)\Bigg|_0^{T_s} \\ &= \frac{T_s}{j2\pi(k_1-k_2)}(1-1) = 0 \end{aligned}$$

Frequency Spectrum

$$s(t)=\sum_{k=0}^{N-1} S_k \underbrace{\frac{1}{\sqrt{N}} 1_{[0,T_s]}(t)\exp\left(j\frac{2\pi kt}{T_s}\right)}_{c_k(t)} \qquad \Delta f = \frac{1}{T_s}$$

This is the term that makes the technique FDM.

$$1_{\left[-\frac{T_s}{2},\frac{T_s}{2}\right]}(t) \xrightarrow{\mathcal{F}} T_s \operatorname{sinc}(\pi T_s f)$$

$$c(t)=\frac{1}{\sqrt{N}}1_{[0,T_s]}(t) \xrightarrow{\mathcal{F}} C(f)=\frac{1}{\sqrt{N}}T_s e^{-j2\pi f\frac{T_s}{2}}\operatorname{sinc}(\pi T_s f)$$

$$c_k(t)=c(t)\exp\left(j\frac{2\pi kt}{T_s}\right) \xrightarrow{\mathcal{F}} C_k(f)=C\left(f-\frac{k}{T_s}\right)=C(f-k\Delta f)$$

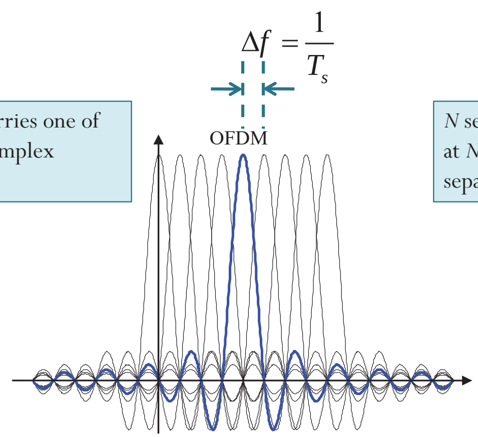
$$\begin{aligned} s(t)=\sum_{k=0}^{N-1} S_k c_k(t) \xrightarrow{\mathcal{F}} S(f) &= \sum_{k=0}^{N-1} S_k C_k(f) \\ &= \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} S_k e^{-j2\pi(f-k\Delta f)\frac{T_s}{2}} T_s \operatorname{sinc}(\pi T_s(f-k\Delta f)) \end{aligned}$$

Subcarrier Spacing

$$s(t)=\sum_{k=0}^{N-1} S_k \frac{1}{\sqrt{N}} 1_{[0,T_s]}(t)\exp\left(j\frac{2\pi kt}{T_s}\right)$$

$$S(f)=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} S_k e^{-j2\pi(f-k\Delta f)\frac{T_s}{2}} T_s \operatorname{sinc}(\pi T_s(f-k\Delta f))$$

Each QAM signal carries one of the original input complex numbers.

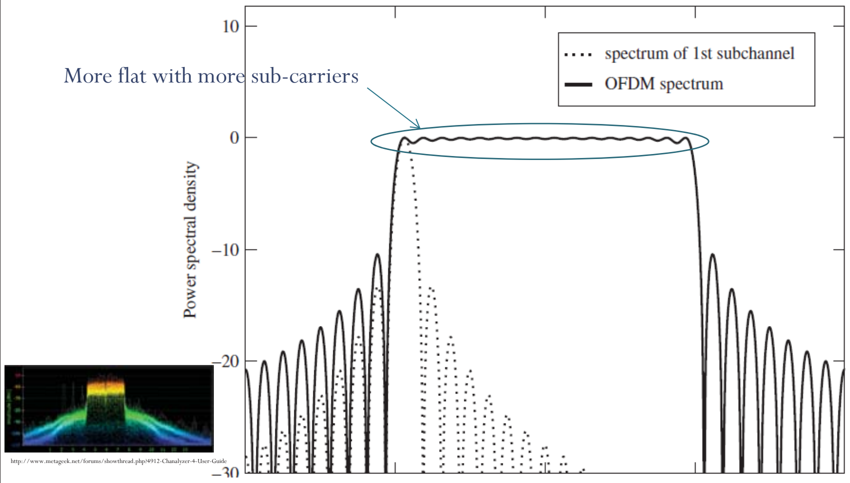


N separate QAM signals, at *N* frequencies separated by the signaling rate.

The spectrum of each QAM signal is of the form with nulls at the center of the other sub-carriers.

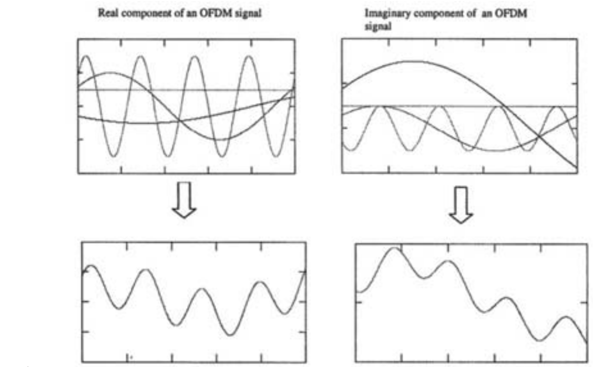
Spectrum Overlap in OFDM

Normalized Power Density Spectrum



[Fazel and Kaiser, 2008, Fig 1-5]

Time-Domain Signal



[Bahai, 2002, Fig 1.7]

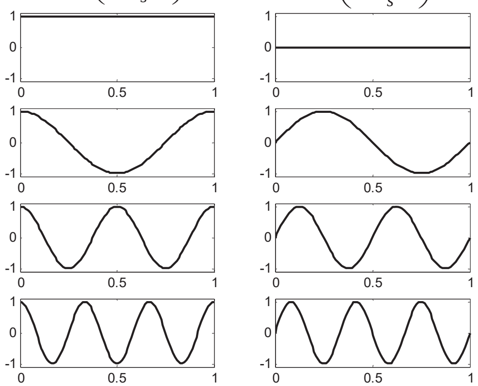
Real and Imaginary components of an OFDM symbol is the *superposition of several harmonics* modulated by data symbols

$$s(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k \exp\left(j \frac{2\pi kt}{T_s} \right), \quad 0 \le t \le T_s$$

$$\mathrm{Re}\{s(t)\} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \left(\underbrace{\mathrm{Re}\{S_k\} \cos\left(\frac{2\pi kt}{T_s} \right)}_{\text{in-phase part}} - \underbrace{\mathrm{Im}\{S_k\} \sin\left(\frac{2\pi kt}{T_s} \right)}_{\text{quadrature part}} \right)$$

OFDM Carriers: N = 4

$\cos\left(\frac{2\pi kt}{T_s}\right)$ $\sin\left(\frac{2\pi kt}{T_s}\right)$



Summary

- So, we have a scheme which achieves
  - Large symbol duration (T_s) and hence less multipath problem
  - Good spectral efficiency
- One more problem:
  - There are so many carriers!

Discrete Fourier Transform (DFT)

Transmitter produces

$$s(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k \exp\left(j \frac{2\pi k}{T_s} t \right), \quad 0 \le t < T_s$$

Sample the signal **in time domain every T_s / N** gives

$$s[n] = s\left(n \frac{T_s}{N} \right) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k \exp\left(j \frac{2\pi k}{\cancel{T_s}} n \frac{\cancel{T_s}}{N} \right)$$

$$= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k \exp\left(j \frac{2\pi kn}{N} \right) = \sqrt{N} \, \text{IDFT}\{S\}[n]$$

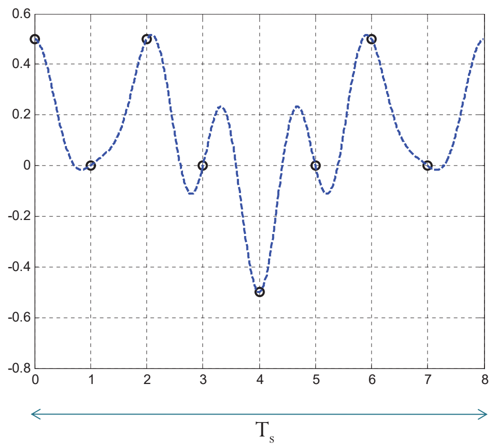
where $\text{IDFT}\{\vec{\mathbf{S}}\}[n] = \frac{1}{N} \sum_{k=0}^{N-1} S_k \exp\left(j \frac{2\pi kn}{N} \right)$

$\vec{\mathbf{S}} = \left(S_0 \quad S_1 \quad \cdots \quad S_{N-1} \right)^T$

We can implement OFDM in the discrete domain!

DFT Samples

$$s(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k \exp\left(j \frac{2\pi kt}{T_s} \right), \quad 0 \le t \le T_s$$



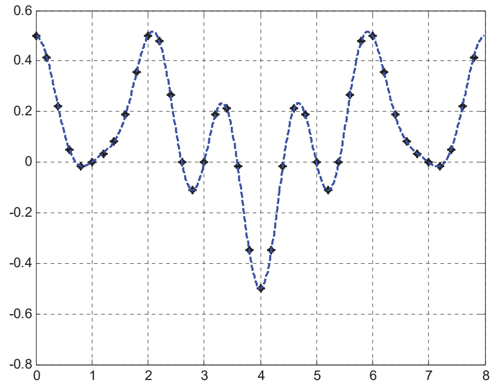
$$s[n] = s\left(n \frac{T_s}{N} \right)$$

$$= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k \exp\left(j \frac{2\pi kn}{N} \right)$$

$$= \sqrt{N} \, \text{IDFT}\{S\}[n]$$

$$0 \le n < N$$

Oversampling



Oversampling (2)

- Increase the number of sample points from N to LN on the interval $[0, T_s]$.
- L is called the **over-sampling factor**.

$$s[n] = s\left(n \frac{T_s}{N} \right), \quad 0 \le n < N \quad \Rightarrow \quad s^{(L)}[n] = s\left(n \frac{T_s}{LN} \right), \quad 0 \le n < LN$$

$$s^{(L)}[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k \exp\left(j \frac{2\pi k}{\cancel{T_s}} n \frac{\cancel{T_s}}{LN} \right) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k \exp\left(j \frac{2\pi kn}{LN} \right)$$

$$= \frac{1}{\sqrt{N}} LN \left(\frac{1}{LN} \sum_{k=0}^{N-1} S_k \exp\left(j \frac{2\pi kn}{LN} \right) \right)$$

$$= L\sqrt{N} \left(\frac{1}{LN} \left(\sum_{k=0}^{N-1} S_k \exp\left(j \frac{2\pi kn}{LN} \right) + \sum_{k=N}^{NL-1} 0 \exp\left(j \frac{2\pi kn}{LN} \right) \right) \right)$$

$$= L\sqrt{N} \left(\frac{1}{LN} \sum_{k=0}^{NL-1} \tilde{S}_k \exp\left(j \frac{2\pi kn}{LN} \right) \right) = L\sqrt{N} \, \text{IDFT}\{\tilde{S}\}[n]$$

Zero padding:

$$\tilde{S}_k = \begin{cases} S_k, & 0 \le k < N \\ 0, & N \le k < LN \end{cases}$$

Scaling

Oversampling: Summary

N points

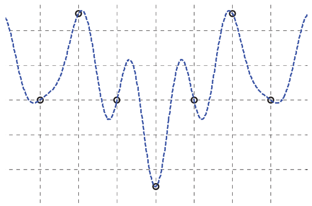
$$s[n] = s\left(n\frac{T_s}{N}\right) = \sqrt{N}\ \text{IDFT}\{\mathbf{S}\}[n]$$
$$0 \le n < N$$

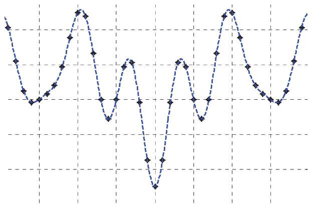
LN points

$$s^{(L)}[n] = s\left(n\frac{T_s}{LN}\right) = L\sqrt{N}\ \text{IDFT}\{\tilde{\mathbf{S}}\}[n]$$
$$0 \le n < LN$$

Zero padding:

$$\tilde{S}_k = \begin{cases} S_k, & 0 \le k < N \\ 0, & N \le k < LN \end{cases}$$





Summary: Three steps towards modern OFDM

1. To mitigate multipath problem
   → Use multicarrier modulation (FDM)
2. To gain spectral efficiency
   → Use orthogonality of the carriers
3. To achieve efficient implementation
   → Use FFT and IFFT

ECS455: Chapter 5

OFDM

5.4 OFDM in LTE

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Advanced Mobile Wirless Systems

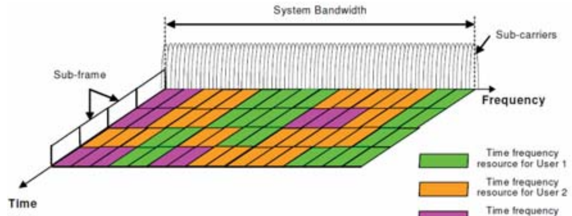
(IEEE) (Ultra Mobile Broadband)

	Mobile WiMAX	3GPP LTE	3GPP2 UMB
Channel bandwidth	5, 7, 8.75, and 10 MHz	1.4, 3, 5, 10, 15, and 20 MHz	1.25, 2.5, 5, 10, and 20 MHz
DL multiplex	OFDM	OFDM	OFDM
UL multiple access	OFDMA	SC-FDMA	OFDMA and CDMA
Duplexing	TDD	FDD and TDD	FDD and TDD
Subcarrier mapping	Localized and distributed	Localized	Localized and distributed
Subcarrier hopping	Yes	Yes	Yes
Data modulation	QPSK, 16-QAM, and 64-QAM	QPSK, 16-QAM, and 64-QAM	QPSK, 8-PSK, 16-QAM, and 64-QAM
Subcarrier spacing	10.94 kHz	15 kHz	9.6 kHz
FFT size (5 MHz bandwidth)	512	512	512

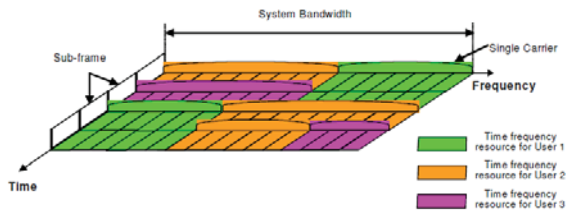
[Myung and Goodman, 2008]

LTE: Multiple Access

- Downlink: **OFDMA** (or we can simply say OFDM)

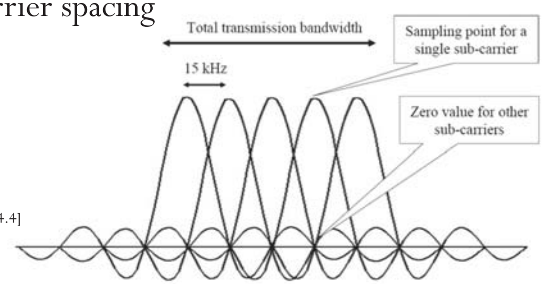


- Uplink: **SC-FDMA**



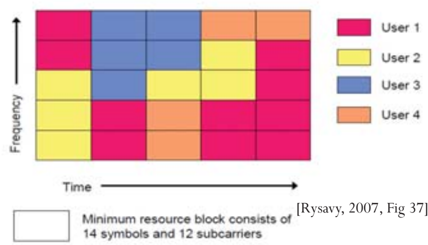
LTE: OFDMA

- 15 kHz subcarrier spacing



[Holma and Toskala, 2009, Fig 4.4]

- Downlink Resource Assignment in Time and Frequency



[Rysavy, 2007, Fig 37]